Answers for class prep quiz on section 4.7, Stewart's Calculus (8th ed.)

1. **Answer:** (c). Again, a procedure that works much of the time is:

   (i) Name variables and translate the facts of the problem into equations.

   (ii) Determine the quantity that needs to be minimized/maximized, and identify any other equations as constraints.

   (iii) Use constraints to reduce the problem to a one-variable min/max problem.

   (iv) Solve the one-variable min/max problem using calculus.

2. **Answer:** (c). The problem asks for the cylindrical container with the largest possible volume, so we are trying to maximize $V = \pi r^2 h$. Therefore, the fact that the surface area of the cylinder is required to be 50 square inches must be a constraint, so our (only) constraint is $2\pi r^2 + 2\pi r h = 50$. Oh, and if you're curious about how the problem continues from there, the constraint allows us to solve for h:

$$2\pi r h = 50 - 2\pi r^2$$
$$h = \frac{25}{\pi r} - r$$

   So our problem boils down to maximizing the function

$$V = \pi r^2 h = V = \pi r^2 \left(\frac{25}{\pi r} - r\right) = 25r - \pi r^3 = f(r).$$

3. **Answer:** (b). Let w and ℓ, respectively, be the width and length of our rectangle. We want to minimize the perimeter $p = 2w + 2\ell$, given that the area $A = \ell w = 24$. Therefore, $\ell w = 24$ is our constraint, and we must have $\ell = \frac{24}{w}$. Our problem therefore boils down to minimizing

$$p = 2w + 2\ell = 2w + 2\left(\frac{24}{w}\right) = f(w).$$

4. **Answer:** (c). To start by translating the problem, "What is the point on the line $y = 5x + 7$" translates to "Find (x, y) such that $y = 5x + 7$," and "closest to the origin" translates to "with **minimum** distance to the origin." The distance from a point (x, y) to the origin is

$$d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}.$$

So we want to minimize $d = \sqrt{x^2 + y^2}$ given the constraint $y = 5x + 7$. Note that instead of minimizing d, we can minimize $d^2 = x^2 + y^2$, making our computation much cleaner. Applying the constraint $y = 5x + 7$, we see that we need to minimize

$$d^2 = x^2 + y^2 = x^2 + (5x + 7)^2 = 26x^2 + 70x + 49 = f(x).$$

Solving $0 = f'(x) = 52x + 70$, we get $x = -\frac{35}{26}$. The first derivative test shows that the global minimum of $f(x)$ occurs at $x = -\frac{35}{26}$, so the point we want is at $x = -\frac{35}{26}$ and

$$y = 5\left(-\frac{35}{26}\right) + 7 = \frac{-175 + 182}{26} = \frac{7}{26}.$$